**STRAIN ENERGY**

Strain Energy = Stored energy due to deformation

= Work done for deformation (Within Elastic limit/Proportionality limit)

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| **“Strain Energy is capable to doing some work”**  **Resilience:** Strain energy stored within elastic limit.  UR = [ Aσ-ε] \* Volume  = (1/2) σ ε \* V  By hook’s Law,  UR = (1/2) (σ2V / E)  **Proof Resilience:** MaximumStrain energy stored up to elastic limit.  UPR = (1/2) (σ2 V / E), here σ = Stress at elastic limit  **Toughness:** Strain energy stored up to Fracture.  Toughness is maximum amount of shock energy absorbed before fracture.  Toughness is useful for while designing accidental over loading. |  |

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| **Strain Energy / Volume** | |
| **Modulus of Resilience** | **Modulus of Toughness** |
| (Strain Energy/Volume) up to elastic limit/ Proportionality limit | (Strain Energy/Volume) till fracture |
| [ Aσ-ε] up to Proportionality limit | [ Aσ-ε] till fracture |

**Can we change resilience and toughness?** Yes, By Alloying we can change property.

**Toughness** depends on **Strength** and **ductility.**

Eg.Hard Steel (0.6%C), Soft Steel (0.1%C), Structure Steel (0.2%C) (Moderate Toughness)

**Young’s Modulus remains constant when alloying material.**

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| **Strain Energy Stored due to following loading** | | | |
| Axial Loading | Bending | Torsion | Shear |

**Strain Energy Due to Axial Loading:**

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| **For Gradually Applied Load,**  Work Done = Work Stored  (1/2) P δL = (1/2) σ ε \* Volume  From Young’s Modulus,  σ = P / A  UG = (1/2) (P2V / A2E) |  |  |
| **For Suddenly Applied Load,**  Work Done = Work Stored  P δL = (1/2) σ ε \* Volume  From Young’s Modulus,  σ = 2P / A  US = 4 UG |  |  |
| **For Impact Load,**  Work Done = Work Stored ===> P (h + δL) = (1/2) σ ε \* Volume  From Young’s Modulus,  σ = σG \* Impact Factor, here σG **=** Stress at Gradually Applied  δL can be found from Young’s modulus equation. | |  |

U = (1/2) (σ2 V / E), here σ put from above derivation.

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| **Strain Energy Stored due to following loading (Gradually Applied)** | | | |
| **Axial Loading** | **Bending (For Long Beam)** | **Torsion** | **Shear (For Short Beam)** |
| U = | U = , | U = , | U = |
| U = (1/2) P δL | U = (1/2) M θ | U = (1/2) T θ | U = (1/2) τ ϕ |

**Castigliano’s Theorem:**

If an elastic Structure is in equilibrium under the action of different forces (Pi where i = 1,2,….)

**Theorem 1:**

The Partial Derivative of Strain Energy with respect to the **point load** is a deflection of a structure **at the point** of application of load **in the direction** of applied load.

∂U / ∂Pi = δi

**Theorem 2:**

The Partial Derivative of Strain Energy with respect to **concentrated bending moment/ point couple** is the slope of a structure **at the point** of application of moment **in the sense** of applied load.

∂U / ∂Mi = θi

**Tip:**

1) If no load present at the desired location to find deflection, then add dummy load ‘Q’ find,

δQ = ∂U / ∂Q || (Q=0)

2) If no moment present at the desired location to find slope, then add dummy moment ‘MQ’ find,

θQ = ∂U / ∂MQ|| (MQ =0)